

Element-Free Galerkin Method for Electromagnetic Field Computations

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Although numerically very efficient the finite element method exhibits difficulties whenever the necessity of remeshing of the analysis domain must be performed either due to large changes in the geometry, deformations or movements. For such problems utilizing meshless computation methods is very promising. In this paper, a kind of meshless method called the element-free Galerkin method is introduced for electromagnetic field computation. The mathematical background for the moving least square approximation employed in the method is given, and the numerical implementation is briefly discussed. Application of the proposed method for electromagnetic field computation and verification of the obtained results using theoretically known solution is also presented.

Key Words: Element-free Galerkin method, Moving Least Square Methods, Finite Element Methods, Electrostatic analysis.

1 Introduction

Over the past two decades, the finite element method has been established as a very powerful numerical technique for solving various engineering problems. Its main power results from the fundamental idea of replacing a given continuous function defined over the entire analysis domain by piecewise approximations, usually polynomials, over a set of finite number of geometrically simple domains called finite elements. Although the idea of domain division is really ingenious, meshing of the analysis domain can sometimes be very laborious and time consuming. Moreover, there are certain classes of problems for which the finite element method is difficult, or even impossible to be applied. For example, in electromagnetic field computation, problems which involve large geometrical changes or deformations of an analysis model such as inverse shape optimization, melting and metal casting, moving

conductors, cracks, etc., finite element method usually requires remeshing in order to insure equality between finite element boundaries and the moving discontinuities. Therefore, not only decreasing of the accuracy, but the huge computation time can be experienced during numerical analysis. For such problems, development of advanced methods which do not rely on meshing are very advantageous.

The main objective for the development of meshless methods is to eliminate even to a small degree the complexity of the analysis method by making approximation which is entirely based in terms of nodes, not elements. Thus, by utilizing these methods it becomes possible to enlarge the area of applications without very laborious and time consuming remeshing process.

In this paper, for the first time the authors present successful implementation of the the element-free Galerkin (EFG) method [1] for electromagnetic field computations. First, the basic idea and the mathematical background for the method are given. Next, the numerical implementation of the method is addressed. Finally, an applica-

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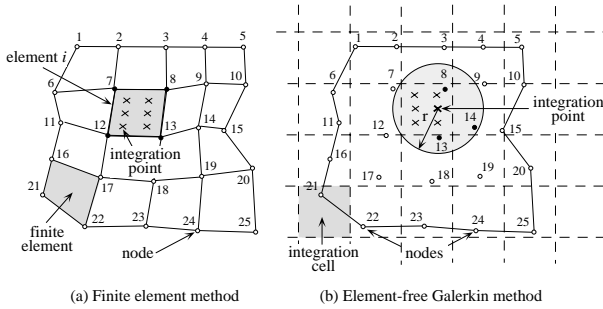


Fig. 1: Finite element method vs. Element-free Galerkin method

tion of the EFG method for a simple 1-D model with known exact solution is presented and the numerical results are compared with the analytical solution. The accuracy of the results and the convergence of the EFG method is also briefly discussed.

2 Element-free Galerkin Method

The element-free Galerkin (EFG) method is considered as a meshless method, because to obtain an approximated solution it requires only the definition of a set of nodes distributed over the entire analysis region and the definition of the boundary conditions [2]. However, for numerical evaluation of the integrals generated using Galerkin procedure, some kind of mesh must be considered. In the EFG, this mesh does not depend on the node disposition and it is far simpler than the ordinary finite element mesh. This, so called integration mesh is usually simple rectangular mesh which overlaps the entire analysis region as shown in Fig. 1. In the finite element analysis the integration is performed at several integration points inside each finite element, and the element matrix is defined by the element nodes, e.g. in Fig. 1(a) nodes: 7, 8, 12 and 13. On the other side, in the EFG method the integration is performed at each integration point of a simple integration cell. The domain of influence of each integration point is defined by the radius of influence r , as shown in Fig. 1(b). Therefore,

for each integration point a list of nodes that lies inside the domain of influence is indispensable. For example, a list of nodes for integration point shown in Fig. 1(b), consists nodes 8, 13 and 14. We have to point out that that the size and number of these integration cells together with the number of integration points for each integration cell have significant influence over the accuracy of the results and computation time.

2.1 Moving least square (MLS) approximation

For approximation of the unknown function, the EFG utilizes the moving least square (MLS) method which encompasses the following three components: a weight function associated to each node, a polynomial basis, and a set of coefficient which depend on the position. In the MLS approximation, the interpolant function $u^h(\mathbf{x})$ of the unknown function $u(\mathbf{x})$ inside the domain Ω is defined by [1]

$$u^h(\mathbf{x}) = \sum_{i=1}^m p_i(\mathbf{x}) a_i(\mathbf{x}) \equiv \mathbf{p}^T(\mathbf{x}) \mathbf{a}(\mathbf{x}), \quad (1)$$

where m is the number of terms in the basis, $p_i(\mathbf{x})$ are the monomial basis functions, and $a_i(\mathbf{x})$ are their coefficients which are functions of the spatial coordinates \mathbf{x} . A common linear basis in 2-D space is

$$\mathbf{p}^T = \{1, x, y\}, \quad (2)$$

Coefficients $a_i(\mathbf{x})$ can be obtained by performing a weighted least square fit by minimizing the difference between the local approximation and the function which yields to the following quadratic form

$$J = \sum_I w(\mathbf{x} - \mathbf{x}_I) \left[\sum_i p_i(\mathbf{x}_I) a_i(\mathbf{x}) - u_I \right]^2, \quad (3)$$

where $w(\mathbf{x} - \mathbf{x}_I)$ is a weighted function [3].

3 Application

The usefulness of the EFG method was verified using a simple 1-D electrostatic model with

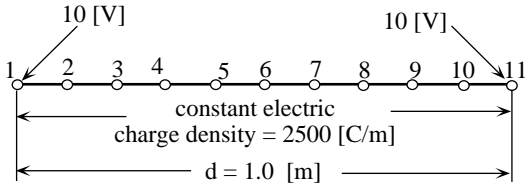


Fig. 2: 1-D test model.

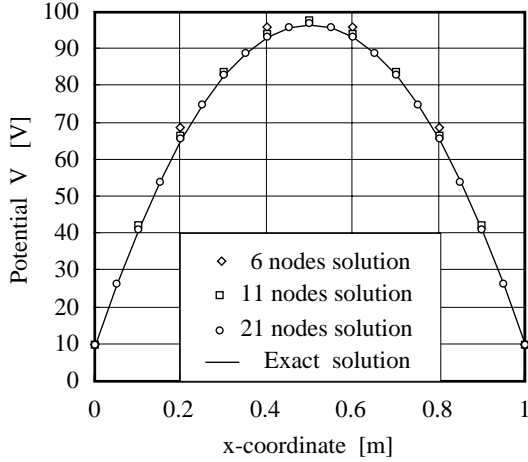


Fig. 3: Exact solution vs. computational results.

known exact solution which is given in Fig. 2. It has constant electric charge density $\rho = 2500[C/m]$, and constant relative permittivity $\epsilon_r = 3.6$. Three runs were performed in order to obtain the convergence characteristic of the EFG method: for model with six, eleven and twenty-one nodes with constant number of integration cells equal to ten and two integration points in each cell. The obtained solutions together with the exact one are given in Fig. 3. The relative error for different number of nodes is given in Table I.

From the obtained results it is apparent that the EFG method provides very accurate results

Table 1: Accuracy of the results vs. number of nodes for Model I

	6 nodes	11 nodes	21 nodes
Maximum error (%)	3.41	1.21	0.139

s even with a small number of nodes and that the results converge very fast with increasing the number of the nodes. This opens a new perspective for the EFG method as an adaptive technique where by simply dropping a large number of nodes in the area where a large gradient of the unknown function is expected the accuracy of the results could be significantly increased. Finally, we like to point out that from computational point of view, it is advisable to increase the accuracy of the results by increasing the number of cells and keeping the number of integration points to the modest level, instead of using large integration cell with large number of integration points.

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